CORRELATION BETWEEN STRUCTURAL ENERGY AND THE PROPERTIES OF THE NATURALLY RADIOACTIVE NUCLEI

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ABSTRACT. The deviations of the experimentally obtained binding energy of the nuclei from that given by the Bethe-Weiszacker relation have been studied for the α and β active nuclei in the natural radioactive range In a previous work (Dutta and others, 1962) these deviations have been considered to be the structural energy of the nuclei, according to the generally accepted ideas It is observed from a graphical representation that the α and β energies, as also their half lives, are intimately connected with the change 'S', in the structural energy, from the disintegrating to the product nuclei Suitable relations in terms of the known constituents of the nuclei and the structural energy change 'S' have been derived for both the α-energy values and the observed β (maximum) energy values They are given by the relation,

$$E_{\alpha}(\text{obs}) = S + .1704(3Z-N)-17.61 \text{ Mev.}$$

$$E_{\beta}(\text{obs}) = S\left(1+0.25\frac{dS}{dN}\right)-0.358(2Z-N)+12.82 \text{ Mev.}$$

The agreement with observed values are quite satisfactory.

It is known that the α-energy values for different nuclei do not come down lower than a value of the order of 4 Mev. The formation of the α-particle in the radioactive nucleus is also not properly understood. Based on a scheme for the formation of the α-particle, a relation has been obtained to determine the lower limit of α-energy values for different nuclei in the form

$$E_{\alpha}(\text{lim}) = -8\times(\text{Binding energy of the product nucleus per nucleon})-56.6. \text{ Mev.}$$

The observed α-energy values are generally higher than this lower limit, as expected.

Further, the half lives of all the even-even and odd-odd nuclei have been related to a function of E_{α}, Z, N such that they can be calculated with fair agreement. We have for these nuclei,

$$\log\frac{1}{\tau} = 2\ 117[N-Z)^{3}\{E_{\alpha}^{1/2}+b_{i}(90-Z)\}+C_{i}(50-\overline{N-Z})]-69\ 54$$

where b = 018 for even-even nuclei, 009 for odd-odd nuclei, C_i is of the order of 1.7. It gives fair agreement between calculated and observed values.

INTRODUCTION

In a previous paper (Dutta and *et al.*, 1962) we have studied the deviations of the binding energy of all stable nuclei, calculated by the Bethe-Weiszacker relation, from the experimentally determined values, as tabulated by Everling and others (1960). The significant findings have been discussed there. We have now extended this study beyond the stable nuclei of lead and bismuth, to cover the naturally radioactive group, as also the artificially radioactive nuclei in this region. The complete series $4n$, $4n+1$, $4n+2$ and $4n+3$, as shown in the schematic representation, in Fig. 1, where the full lines indicate naturally radioactive series, as also a large number of other artificially radioactive nuclei have been covered up in the present study. We have covered in all, about 80 α-disintegrat-

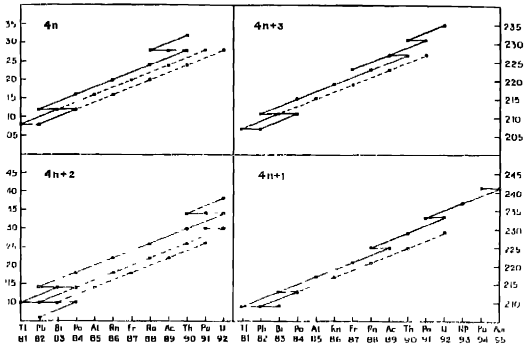

Fig. 1. Schematic representation of the radioactive series.

ing nuclei for which binding energy data are available and 25 β-disintegrating nuclei, for consideration. Their binding energies have been calculated from the Bethe-Weiszacker relation,

$$E = -a_1A + a_2A^{2/3} + a_3Z^2/A^{1/3} + a_4(N-Z)^2/4A \tag{1}$$

We have utilised the constants as determined by us, and discussed in our previous paper. They are tabulated again, along with those by Green (1954) in Table 1.

TABLE I

Constants for the Bethe-Weiszacker relation

	a_1	a_2	a_3	a_4
Dutta and others	16.719	18.505	0.751	96.856
Green	16.918	19.120	0 763	101.78

The deviations of the experimental binding energy values as tabulated by Everling, from those calculated by the Bethe-Weiszacker relation, have been calculated for all these nuclei. We have plotted and connected them through α-disintegration processes for the natural radioactive series only, in Fig. 2, in order to avoid confusion by the intersection of lines due to different series. The nature and characteristics of the curves due to other disintegration series have been found to be similar, in all respects. We have plotted in the curve the ΔE values for α-disintegrating nuclei against mass numbers, as followed in the previous paper. To bring out the β-disintegration characteristics to better relief, one should plot the Δ E values against neutron numbers, as we have done later.

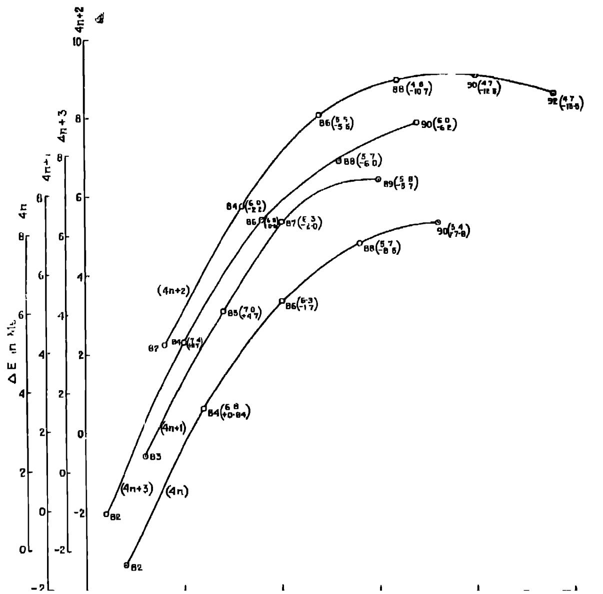

Fig. 2 ΔE Vs. mass numbers of the α-disintegrating nuclei of the four radioactive series

It will be observed from the graph that the change in the Δ E values from one nucleus to its disintegration product, which we prefer to call as S, the structural energy change, varies with the α-energy as also with the half lives τ in a very regular way. The structural energy change, S, is, thus, intimately related with the energy of the α-particles and through it to the half lives of disintegration.

In the α-disintegration chain, in a particular series, the values of S change from a small negative value, through a nearly vanishing magnitude, to a comparatively large positive value. The α-energy values and the half lives, correspondingly suffer a systematic change, starting from apparently arbitrary positive values. This is true for all the different series, although the magnitude of the energy and half life values, in relation to the structural energy change S, varies with the series.

In the case of the β-energies, the maximum energies observed and the structural energy change S, do not appear to be immediately correlated. We will, however, take up the problem of observed β-energy maximum, in relation to the structural energy change, in a later section. We have utilised the data for the α-energies and the maximum β-energies given in the National Bureau of Standard, Circular No. 499, 1950 and in the Table by Strominger and others (1958).

ANALYSIS OF α-PARTICLE ENERGY

To analyse the data, one observes that in the case of almost all the α-disintegrating nuclei, the energy of the α-particle satisfies a simple relation namely,

$$E'-E'' = E_\alpha+(\mathrm{B}.E)_\alpha+\text{recoil energy} \quad \dots \quad (2)$$

Here E' and E'' are the experimental binding energies of the disintegrating and the product nuclei, obtained from Everling's table and the binding energy of the α-particle is taken to be 28 3 Mev The recoil energy comes to a magnitude of the order of 0.1 mev., and may be neglected generally. For different associated α-energies or energy spectrum for one nucleus, one should have different binding energies, for which, unfortunately, the binding energy data are not available yet. We have associated the binding energy data with the most abundant process of disintegration of a particular nucleus. The above relation originates from the fundamental mass and energy balance principle. Thus :

$$E' = M'-\Sigma(m'_n+m'_p); \quad E'' = M''-\Sigma(m''_n+m''_p)$$
$$(B.E)_\alpha = M_\alpha-\Sigma(m_n, m_p)_\alpha$$

giving us, from relation (2),

$$\mathrm{M}' = M''+M_\alpha+E_\alpha+\text{recoil energy of } E_\alpha \text{ product nucleus}, \quad \dots \quad (2a)$$

the energy balance equation. The α-particle energies, thus, satisfy the fundamental energy principle rigidly. It is determined by the binding energies of the disintegrating and the product nuclei, which remain completely arbitrary, so far.

To proceed to understand the energy relationship more explicitly, we may put E' and E'' in terms of the Bethe-Weiszacker binding energies E'_B and E''_B along with the structural energies $\Delta E'$ and $\Delta E''$. It gives us

$$E'-E'' = S+E'_B-E''_B$$

where 'S' is the structural energy change $\Delta E'_B-\Delta E''_B$, measured in mev., from the disintegrating to the product nuclei. Thus, in view of relation (2), we may put the α-energy relation, in the form,

$$E_\alpha = S+E'_B-E''_B-(B.E)_\alpha \text{ a recoil energy} \quad \dots \quad (2c)$$

The involved terms, beside the structural energy change S, is rather complicated to be interpreted in terms of simple nuclear characteristics. One may proceed to find a more easily understandable relation by tabulating the composition characteristics of the complete set of different α-disintegrating series, separately, along with their $(E_\alpha - S)$ values. It becomes evident that the $(E_\alpha - S)$ values for a particular series, with the fixed value of $(N-Z)$, increase with $(N+Z)$ or the nucleon number of the disintegrating nucleus and for different series, with any fixed value of $(N+Z)$, $(E_\alpha - S)$ values, similarly, decreases regularly with $(N-Z)$ values. These observations may be put in a general form of relationship, covering all the α-disintegrating nuclei, as

$$E_\alpha = S + 1704(N+Z-2\overline{N-Z}) - 17.61 \text{ Mev.}$$

or

$$E_\alpha = S + 1704(3Z-N) - 17.61 \text{ Mev.} \quad \ldots (3)$$

The contribution of Bethe-Weiszacker binding energy etc. to the α-energy, then, reduces to be proportional to the difference between the mass number and double the excess-neutron content of the disintegrating nucleus, together with a constant.

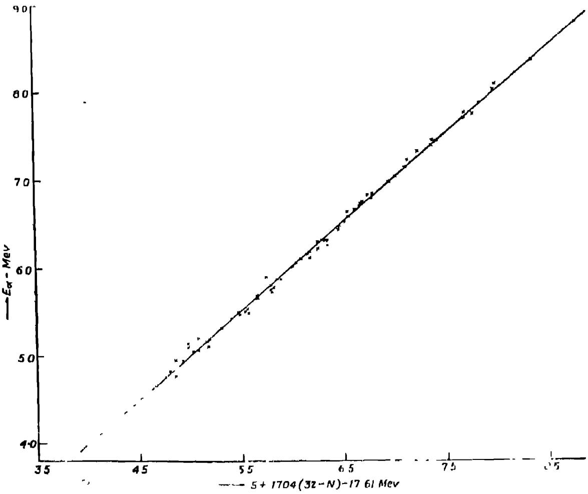

Fig. 3 **α-Decay energy- experimental vs. calculated.**

The agreement between the relation and the observed α-energies is remarkably well, as will be evident from the graphical representation in Fig. 3, where we have plotted the experimental E_α values against the right hand expression for the corresponding disintegrating nuclei, having particular N and Z values and the structural energy change S, as obtained, from $(E' - E'')$. The calculated and observed values of E_α along with the calculated S values, are also tabulated in Table II with the observed and calculated values of log $1/T$ for some nuclei, discussed later on. There is a slight irregularity of the order of 0.1 Mev, for nuclei with low neutron content, like the polonium isotopes ($Z = 84$, $N = 126$, plotted in Fig. 3 on the lower regions of the graph), which may be due to experimental error in the determination of binding energy or the α-energy, or might be due to some other factor not taken into consideration for these low neutron content nuclei.

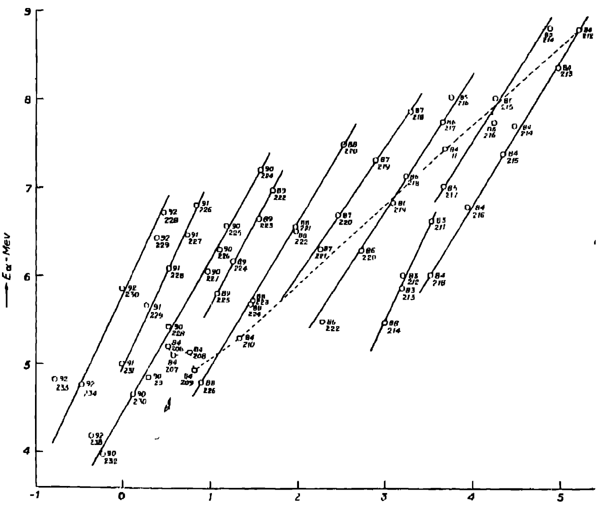

Fig. 4. α-Decay energy against change in ΔE, i.e. 'S' joined through radioactive isotopes.

It is further observed that for any particular element, characterised by the charge value, S increases with the decrease of the neutron number N in the nuclei more or less linearly and for the different elements they are displaced laterally, with some changes in the inclinations of the lines, as will be observed from Fig. 4. For the isotopes with a comparatively small number of neutrons, particularly, for the element polonium ($Z = 84$), the plot of the points give an irregular fall and a rise. The whole character of the diagram is very similar to the well-known α-energy mass number plot of Perlman and others (Perlman, Ghiroso and Seaborg, 1950). A plot of E_α against S or against mass number is, therefore, not very helpful in elucidiation of the determining factor of α-energy. It shows, however, that for the isotopes of an element, the α-energy values increase regularly with the structural energy change S, which increases with gradually lower neutron content of the isotopes except for the very low neutron content elements of the polonium nuclei The plot of Perlman and others moreover shows that a particular value of Z, E_α varies irregularly with the neutron number and for different values of Z the nature of irregularity varies without any possibility of smooth correlationship. It indicates that E_α cannot be measured in terms of any smooth function of N and Z, as has been attempted by G P. Dube and L S. Lingh (1954), Y. P. Varshni (1956), M. K. Ramaswamy (1956), and Evans (1955).

TABLE II

Observed and calculated characteristic of -disintegrating nuclei

$N-Z$	$Z;A$	S	Obs E_α	Calc. E_α	Obs $\log \frac{1}{T}$	Calc $\log \frac{1}{T}$	$N-Z$	$Z;A$	S	Obs E_α	Calc E_α
54	92;238	−0 37	4 18	4 17	−17 15	−17 41	51	95.241	−0 49	5 48	5 58
								94.239	−0 57	5 16	5 16
52	94,240	−0.39	5 10	5 19	−11.28	−11 88		93,237	−0 53	4 77	4 86
	90 232	−0 23	3 98	3 97	−17 70	−17 56		92,235	−0 51	4 56	4 55
50	96;242	−0 42	6 11	6 17	− 7 11	− 7.07	49	95.239	−0.60	5 77	5 82
	94.238	0 36	5 51	5.54	− 9 45	− 9 34		93.235	0 66	5.06	5 07
	92 234	−0 47	4 76	4 76	−12 87	−12 76		92.233	− 0 59	4 82	4 80
	90.230	+0 12	4 66	4 66	−12 50	−12 42		91.231	−0 03	5.00	5 02
	88;226	+0 89	4 80	4.75	−10 70	−10.61		90,229	+0 31	5 05	5.02
	86.222	+2 30	5 49	5 48	− 5 51	− 5 61		89;227	+0.55	4 94	4 92
	84,218	+3 52	6 00	5 99	− 2.26	− 1 85					
							47	93.233	−0 52	5 53	5 56
48	96.240	−0 58	6.30	6 35	− 6 36	− 5 94		91;229	+0 27	5 66	5 66
	94.236	−0 14	5 75	5 81	− 7 93	7 73		90;227	+0 97	6 05	6 03
	92-232	−0 25	5 31	5 32	− 8 98	− 9 07		89.225	+1 09	5 80	5 80
	90.228	+0 52	5 42	5 10	− 7 78	− 7.64		88.223	+1 48	5.72	5 81
	88 224	+1 46	5 68	5 66	− 5 50	− 5 53		87.221	+2 24	6 30	6 30
	86.220	+2 74	6 28	6 26	− 1.74	− 1 80		86.219	+3 10	6 82	6 80
	84.216	+3.95	6 77	6.79	+ 0 84	+ 1 26		85.217	+3 68	7 02	7.03
								81,215	+4 36	7 36	7 36
46	94.234	−0 33	6 20	6 26	− 4 49	− 5 09					
	92;230	−0 01	5 86	5 89	− 6 26	− 5 80	45	92;229	+0.39	6 42	6 46
	91.228	+0 53	6 09	6 09	− 4.90	− 4.29		91.227	+0 75	6 46	6.47
	90;226	+1 11	6 30	6 34	− 3 27	− 3 24		90;225	+1 18	6 57	6 56
	89;224	+1 27	6 17	6 16	− 3.56	− 3 53		89.223	+1 57	6 64	6.63
	88;222	+1 99	6 51	6 54	− 1 58	− 1 59		88.221	+1 98	6.71	6 69
	87:220	+2 47	6 69	6 67	− 1 40	− 1 09		87.219	+2 91	7 30	7 24
	86.218	+3 25	7 12	7 12	+ 1 72	+ 1 54		86;217	+3 68	7.74	7 70
	85.216	+4 27	7 72	7 78	+ 3.52	+ 3.26		85;215	+4.28	8 00	7 98
	84.214	+4.50	7 68	7 69	+ 3 83	+ 4 40		84.213	+5.01	8.34	8.33
44	92;228	+0 47	6 72	6 71	− 2 75	− 2 80	43	84;211	+ 3.71	7 43	7 39
	91.226	+0 85	6 81	6 76	− 2 01	− 2 00					
	90:224	+1 59	7 20	7 15	− 0	− 0.49	42	84;210	+1 33	5 30	5 19
	89,222	+1 73	6 96	6.96	0 74	− 1 17	41	85.211	+1 39	5 89	5 76
	88.220	+2 54	7 49	7 42	+ 1.52	+ 1.09		84.209	+0 82	4 95	4 85
	87;218	+3.29	7 85	7 84	+ 2 30	+ 2 14	40	86;212	+1 30	6 17	6.18
	86.216	−3 7	8 07	7 99	~ + 4 00	+ 3 64		84;208	+0 77	5.14	4 97
	85;214	+4.9	8 78	8 78	~ + 5 70	+ 5.37	39	84.207	+0 59	5 10	4 97
	84.212	+5 25	8.77	8 75	+ 6 70	+ 6.46	38	87;212	+0.75	6 25	6.35
								84.206	+0 53	5 20	5.07

We have already noted that the α-particle energies satisfy the relationships (2) and (3) namely, $E'-E'' = E+(\mathrm{B.E})_\alpha$ or

$$E_\alpha = S+.1704(\overline{N+Z}-2\overline{N-Z})-17.61 \text{ Mev.}$$

With the help of these two relations, it is possible to probe into the reason for the limitation of the α-energy magnitude on the low energy side . In obtaining the lower limit of observed α-energy and in attempting to understand the formation of α-particles, we have departed from the idea that the α-particles cross the central potential barrier and considered that the outcoming particle interacts strongly with the rest of the nucleus as was suggested by Chang (1946) and modified by Preston (1951 and discussed theoretically by Brussaard and Talhoek (1958). It is well known that the energy of the α-particles are limited, on the lower side, to a magnitude of about 4 Mev only. This property should be related to the other emission characteristics of the nuclei These are the observed facts that some of these nuclei are always α-disintegrating, some β-disintegrating, some are both α- and β-disintegrating and finally, the end products of these disintegrating series are stable In our attempt to understand these characteristics on some basis, we note that the scope of the validity of the principles should be considered restricted to the region of the heavy nuclei, in the first instance. Appropriate modifications in other ranges of artificial disintegration are likely.

Let us now consider E' to be the binding energy of the 'A' nucleons in the disintegrating nucleus and E'' as the binding energy of the (A–4) nucleons in the product nucleus for an α-disintegration process The binding energy per nucleon, in this naturally radioactive region is of the order of − 7.7 Mev , and it varies from nucleus to nucleus. If the binding energy per nucleon of the disintegrating and the product nucleus obtained the same value as −7 7 Mev , we would have $E'-E'' = -30.8$ Mev. From the expression for E_α, in relation (2), we would have its magnitude as, $E_\alpha = -30.8$ Mev. $-(B\,E)_\alpha = (-30\,8+28\,3)$ Mev $= -2.5$ Mev., which is a negative quantity and would restrict the formation and emission of the α-particle We now modify the binding energy of the disintegrating nucleus in such a way, that compared to the binding energy of the product nucleus, on the basis of − 7.7 Mev per nucleon, there is a total shortage of energy amounting to 2 5 Mev., in the disintegrating nucleus. The distribution of nucleons in the disintegrating nucleus would, on occasions, take the form of the product nucleus for (A–4) of the nucleons, with its necessary energy distribution, leaving the shortage of 2 5 Mev . to the remaining 4 nucleons. This is plausible, as the (A–4) nuclear composition is known to have a structural configuration, corresponding to that binding energy, when left to itself. The four nucleons, with a shortage of 2.5 Mev , would be left with −28.3 Mev the amount required for the binding energy of the α-particle in its free state. The α-particle, however, could not be formed under the conditions. The available energy −28.3 Mev., would be distributed among the four nucleons, mutually, as also between these nucleons and the neighbouring nucleons of the main structure. The formation of an α-particle, under the conditions, would require that the binding energy per nucleon of the 4-particle entity, with its neighbouring nucleons in the composite structure, is less than the α-particle integration energy per nucleon, as a separate entity, amounting to -7.075 Mev., otherwise the formation of the α-particle would not be possible.

To clarify and illustrate let us take the α-particle in the free state, with the binding energy per nucleon amounting to -7.075 Mev. As one of the alternative processes of conjecturing, this amount may be taken to signify the binding energy of any one of the nucleons to one of its neighbours, in a particular sense or direction, each nucleon behaving in a similar way, in a regular order. In the case of a large close packed structure in space, we may proceed to conjecture in a similar way and are forced to associate a larger number of nucleons, simultaneously, to each nucleon, with a similar behaviour for other nucleons also. Such distribution of binding energies for all the nucleons would compose the complete cohesive character of the larger nucleus.

In a three dimensional close packed structure, each nucleon would be linked with six other nucleons, generally, in the three directions of space, whereas, considering only the α-particle entity in the same structure, we would have each nucleon associated with two other nucleons only, along the two directions of space. The associations of a nucleon with the outside nucleons, corresponding to the larger structure and with the inner group to compose finally the α-particle are thus, in the ratio of 2/3 to 1/3. Further, in the sense that an isolated α-particle, would have each nucleon associated with one other nucleon, in a regular way, we would have in the composite structure, each nucleon associated with three other nucleons in a regular way, to build up the complete cohesional structure. Thus, we may consider, for the possibility of integration of the α-particle as a separate entity from a composite structure, the necessary relationship to be satisfied as,

$$2/3(B.E.)_{p,n} + 1/3(B.E)_{\alpha,n} \gtreqqless -7.075 \text{ Mev}$$

Here, $(B.E.)_{p,n}$ signifies the $(B.E)$ of the product nucleus per nucleon $(B.E)_{\alpha,n}$ is the binding energy available to each of the four nucleons which would form the α-particle and $\gtreqqless$ signifies that the left hand side expression is more positive or weaker in strength. The amount -7.075 Mev. measures the binding energy per nucleon of the α-particle, to be formed finally. As an example, when we have $(B.E)_{p,n} = -7.7$ Mev and $(B.E)_{\alpha,n} = -\frac{28.3}{4}$ Mev, we have the left hand expression amounting to -7.49 Mev. which is much stronger than α-integration energy. The particle, thus, cannot be formed.

We consider here the process already envisaged, that except for the four nucleons to form the α-particle, finally, (A−4) nucleons of the disintegrating nucleus takes the structural form for the product nucleus and the energy difference $E'-E''$ are associated with the four nucleons only, so that,

$$\tfrac{1}{3}(B.E)_{\alpha,n} = 1/12\ (E'-E'').$$

When we put $E'-E'' = E_\alpha+(B.E)_\alpha = E_\alpha-28.3$ Mev,

aaccording to relation (2), we have

$$E_\alpha(\text{lim}) = -8(B\,E\,)_{p,\,n}-56.6 \text{ Mev.} \qquad \ldots \quad (4)$$

where E_α(lim) is determined by the equality relation.

Thus, the lower limit of the emitted α-particle energy may be obtained, on the basis of 3 bonds per nucleon, when the binding energy of the product nucleus is known Calculated on this basis, the lower limit of the emitted α-energy comes out according to expectation, generally, except for nuclei with lower neutron content, where the number of neutrons is equal to or less than 1.5 times the proton number and also for the nucleus (90,232). In these cases the calculated lower limit of the α-energy is higher than the observed energy. It is expected that the number of bonds per nucleon should depend in someway on neutron-proton ratio. We find that the limiting values become adjusted to the proper magnitude, when the bonds per nucleon in these cases of low neutron content nuclei are taken as 2.5 in place of 3, considered for other nuclei. The reasons for disagreement in case of the element (90,232) remain unexplained It is, however, not unlikely that the number of bonds to be associated per nucleon should be gradually adjusted, according to the proportion of neutrons and protons in the nuclei, and thus, the lower limit of the α-particle energy also should be further adjusted by corresponding modifications in the relation (4) above.

When there is no α-disintegration in a nucleus and there is no known nucleus corresponding to the product nucleus composition, we may estimate the expected limiting value of the α-energy from a modified and approximate relationship, which immediately follows from the relation (4) above. Thus, we may put,

$$E_\alpha(\text{lim}) = -\frac{8}{A-4}\cdot[(B.E)_D-\delta E]-56.6 \text{ Mev.} \qquad \ldots \quad (4a)$$

where $(B.E.)_D$ is the binding energy of the disintegrating nucleus and δE is the energy difference in the concerned range, between other nuclear set (Z, A) and $(\overline{Z-2}, \overline{A-4})$, such that $(B.E)_D-\Delta E$ correspond to an approximate value of the binding energy of the product nucleus, making relation (4a) identical with (4). The values of δE are obtainable from binding energy tables.

In Table III below, we have compared the calculated limiting values of the α-energy with the observed energy values for α-disintegrating nucleus or with relation (3), in the form,

$$E_\alpha = S+.1704(\overline{N+Z}-2N-Z)-17.61 \text{ Mev.},$$

which has been found to determine all α-energy values satisfactorily. The limiting α-energy values for all the nuclei, where the neutron number is more than 1.5

TABLE III

Limiting α-energy values and calculated or observed α-energies of various types of heavy nuclei.

Disintegration character of the nucleus	Nucleus	E_α(lim)	E_α(obs) or (calc.)	Remarks
	92,238	4.18	4.2	E_α(obs. or calc.) $> E_\alpha$(lim.)
	92,234	4.43	4.76	
	91,231	4.59	5.0	
	90,227	4.88	6.05	
	90,228	4.84	5.42	
α-disintegrating	86,222	5.26	5.49	
	86,220	5.47	6.28	,,
	86,219	5.55	6.88	
$N > 1.5Z$	85,217	5.74	7.02	
	84,218	5.59	6.0	
	84,216	5.84	6.77	
	84,215	5.93	7.36	
	84,214	6.06	7.68	,,
	84,213	6.19	8.34	
	84,212	6.34	8.78	
	84,211	6.35	7.43	
	90,232	4.53	3.98	E_α obs. $< E_\alpha$ (lim.)
α-disintegrating	87,212	5.9 4.43	6.25	E_α(lim.) with 3 and 2.5 bonds per nucleon. Upper values with 3 bonds.
$N \leqslant 1.5Z$	86,212	6.09 4.57	6.17	
	85,211	6.21 4.68	5.89	
	84,210	6.39 4.80	5.30	,,
	84,209	6.38 4.79	4.95	
	84,208	6.42 4.83	5.14	
	84,207	6.42 4.83	5.10	,,
	84,206	6.45 4.85	5.20	
α and β disintegrating	83,214	6.10	6.07	E_α (calc. or obs.) $\cong E_\alpha$ (lim.).
	83,213	6.32	6.62	
	83,212	5.90	5.56	
	83,211	6.07	6.00	
β-disintegrating	91,234	4.43	$S+4.49$	E_α calculated, except S. S values $-ve$, small in the range
	90,234	4.48	$S+3.80$	
	89,228	4.79	$S+4.15$	
	89,227	4.86	$S+4.31$	
	88,228	4.81	$S+3.47$	
	83,210	6.48	$S+3.13$	E_α calculated, except S. $S \cong 3.5$ in the range.
	82,214	5.96	$S+1.77$	
	82,212	6.18	$S+2.11$	
	82,211	6.31	$S+2.27$	
	82,210	6.40	$S+2.45$	
Stable	83,209	6.40	$S+3.3$ $S-3.3$	Upper values E_α calculated. Lower values E_β calculated. S values strongly negative for both.
	82,208	6.48	$S+2.8$ $S-1.6$	
	82,207	6.50	$S+2.97$ $S-2.03$	
	82,206	6.57	$S+3.13$ $S-3.3$	

times the proton number, have been calculated on the basis of 3 cohesional bonds for each of the neucleus as discussed before. For nuclei with the neutron number 1.5 times or less than the proton number, the limiting α-energy values have been calculated with the cohesional bond number per nucleon as 3 and 2.5 also. The observed α-energy values are in agreement with the limiting values calculated with 2.5 cohesional bonds, as against 3 cohesional bonds required for other nuclei. It shows the necessity of change in bond characteristics with the change in neutron proton proportion.

For the β-disintegrating particles, we may compare the expected α-energy values calculated from the relation (3), with the limiting α-energy values obtainable from the relation (4a), where the S values have to be estimated. It is found in these cases, that the expected α-energy values for such nuclei are lower than the limiting α-energy values. It tends to indicate why these nuclei are not of the α-disintegrating type. For those nuclei which have both α and β disintegrating characteristics, it is observed that the limiting α-energy values and the calcultated ones are nearly equal. For stable nuclei the limiting α-energy values are much higher than the expected ones, calculated by the α-energy relationship. For these nuclei as also for the β-disintegrating nuclei, α-disintegration is debarred by the limiting energy condition.

PRINCIPLES OF α-AND β-DISINTEGRATION,,

This brings us to the problem of β-disintegration. We limit ourselves, in our consideration, to the study of the maximum β-energy of the nuclei, in the naturally radioactive range, along with the artificially radioactive nuclei in the same range. We do not involve ourselves with the continuous character of the β-emission. As in the case of α-energy values, the expected β-emission energy may be calculated by a relation, based on fundamental energy principle, namely,

$$E'-E'' = E_{\beta}+E_{n,p} \qquad \dots \quad (5)$$

where E' and E'' are the binding energies of the disintegrating and the product nuclei and $E_{n,p}$ is the neutron-proton exchange energy, amounting to .78 Mev. As a matter of fact these expected β-energy maxima values have been tabulated by Everling in his table, on the basis of the above relationship. By a process similar to that followed to derive the α-energy expression (3), in terms of N, Z and S, we may find and replace the equation to determine the expected maximum β-energy by the relationship,

$$\left.\begin{aligned} E_{\beta} &= S-0.179\{(N+Z)-3(N-Z)\}+12.28 \text{ Mev.} \\ &= S-0.358\ (2z-N)+12.28 \text{ Mev.,} \end{aligned}\right\} \qquad \dots \quad (6)$$

where S is the structural energy change for β-disintegration. The relation covers all the calculated E_{β} values in this range quite satisfactorily. There is, however, no limiting value of E_{β}, on the low energy side, as it obtains in the case of E_{α}, according to observation. An allowed E_β value in this range is, therefore, given by any positive value for E_β in the expression (6). If we calculate the expected E_β values for the four stable nuclei tabulated in Table II and consider the fact that the expected S values for all of them are negative, they being in the positions of minima for the $\Delta E - A$ curves, we have their expected E_β values strongly negative, indicating stability. The calculated values of E_β are also incorporated in Table III.

We may thus infer the following underlying principles. When the neutron-proton content of a nucleus, along with the structural energy change for a possible disintegration, obtains a value of E_α calculated by relation (3), which is larger than E_α (lim) calculated by relations (4a) or its modified form for the nuclei, with a lower percentage of neutron, we expect an α-particle emission. For the nuclei on the border line having E_α (Calc) $= E_\alpha$ (lim), we obtain both α and β emission. For E_α (Calc) values definitely less than E_α (lim), we have β-emission only and its maximum possible energy can be calculated by relation (6). When the calcualted E_β value becomes negative, the nuclei would belong to the stable category.

RELATION FOR E_β-MAXIMUM

The observed E_β maximum values are not always equal to the values expected from energy consideration, and calculated by the above relations (5) or (6). About fifty percent of the observed values, in this range, agree with the calculated ones, and in some of these cases, there are more than one maximum for one nucleus. Other observed values are generally some fraction of the expected values and in few cases the observed maxima values are definitely larger than the expected ones, on energy consideration. The observed values of β-energy maxima, in this range varies from a vanishingly small magnitude to a value of the order of nearly 4 Mev To understand the discrepancy between the calculated and the observed β-energy maxima, we may plot the ΔE values from the Bethe-Weiszacker relation against neutron numbers, for all the β-active nuclei, in this range, along with the associated α-energy changes, as shown in the figure (5), by full and dotted lines. It would be observed that about 12 β-active nuclei, which have at least one observed value of the same magnitude as the calculated ones, lie on the maxima or minima of the ΔE versus N-graph, which gives us, from the course of the curves, S against N values. These nuclei are, therefore, in stationary states with regard to structural energy change and satisfy the criterion $dS/dN = 0$. The remaining ten nuclei have their observed E_β values as different fractional magnitudes of the calculated ones, while two nuclei have definitely higher observed values. Corresponding to the large or small variations of the observed and calculated values, they lie on a steeply or slowly changing regions of the curves, as will be observed in the figure, having both positive and negative values of dS/dN. These nuclei are on nonstationary states of structure. The large drop in the observed values from the calculated ones for the element 81(Tl), with 127, 128 and 129 neutrons is closely related with their positions in steep regions of the curves $\Delta E - N$. Those nuclei are on rapidly changing states of structure. It is implied that the β-dis-

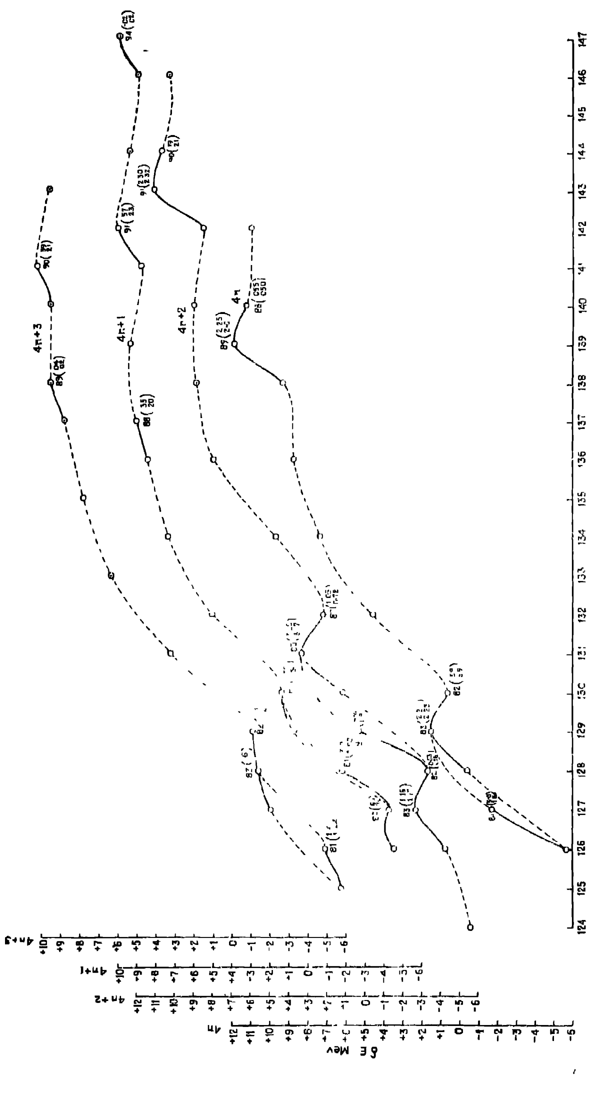

Fig. 5. ΔE Vs. neutron number for α and β-disintergrating nuclei.

integrating nuclei, on such sloping regions of the curve, have not attained the structure required by their constitution fully and should have their structural energy term 'S' in relation (6), accordingly modified It may be noted that from our relationship for S with reference to the curve, dS/dN is negative for a downward slope of the curve in the direction of decreasing N. This indicates the criterion for positive dS/dN values also

We, therefore, replace the structural energy term 'S' in relation (6), by a more suitable term for the observed β-energy values, in accordance with the observations, just made and replace S by $S\left(1+K\dfrac{dS}{dN}\right)$, in the relation, for the observed β-energy A constant value of $K = 0.25$, satisfies the observed β-energy, values of the nuclei, on the basis of the curves as drawn in Fig. (5). We may thus, modify the relation (6), for observed β-energy in form,

$$E_\beta(\text{observed maxima}) = S\left(1+0.25\frac{dS}{dN}\right) - 0.358(2Z-N)+12.82\ \text{Mev.}, \ldots \quad (6a)$$

which obtains all the observed values satisfactorily, including those with observed values higher than the expected or calculated ones. Previous attempts by Glueckauf (1948), Suess and Jensen (1952) and Way and Wood (1954) to systematise the β-energy maxima either observed or expected in this region of heavy nuclei do not suggest definite relationship to calculate the values. It may be noted, however, that there is a scope of adjusting the curvatures of the curves to suit the observed values. This scope of adjustment of the curves gives us also a range of the possible β-energy values in a continuous way, which is significant. It also limits the flexibility in drawing the curves in view of the limitations of the range of observed values for a nucleus.

We may further remark that the observed multiple valued β-energies should correspond to multiple valued binding energies. We have, however, till now, only single binding energies for disintegrating nuclei, and presumably that for the more stationary states, corresponding to the largest of the observed values, with which the curves have been fitted.

It brings us back to the case of calculated α-energies by relation (3), where we have not modified the term S in the expression for E_α, to get good agreement between observed and calculated values, although the nuclei lie on as steep curves as in the case of β-disintegration. This implies that the structural change required by the ΔE curve, for α-disintegration is generally established. The structure evidently should be automatically adjusted to the constitution, as, in such emission the product nucleus is already formed within the disintegrating nucleus, according to our observations in connection with the limiting value of α-particle energies.

HALF LIFE--α-ENERGY RELATIONSHIP

We conclude our work with a study of the interrelation between the half lives and energy of the α-disintegrating nuclei. Quite a lot of work has been done on this line, beginning with Geiger and Nuttal's work (1911), and elaborated initially by Perlman, Ghiroso and Seaborg (1950). A more comprehensive up-to-date information is complied by Preston (1962). The Geiger Nuttal law connecting the ranges of the α-particles with the decay constant λ, came into difficulties because of misfits, and this difficulty has not been removed till now It has been noted by us that as a guiding principle for all α-disintegrating nuclei, one may take log $1/\tau$, as roughly proportional to $E^{\frac{1}{2}}$, for a definite amount of excess neutrons. The slopes are proportional to the reciprocal of the cubes of the excess neutron amount and the displacements along the $E^{\frac{1}{2}}$ scale are also measurable in terms of $(N-Z)$. There are, however, quite a number of large or small deviations.

It is, however, possible to adjust the deviations from a smooth linear course, by suitable terms depending on the nuclear charge, particularly, for all the even-even and odd-odd nuclei. As an example we may refer to the plot of points, in Fig. (6), corresponding to an excess neutron content, amounting to 50, where log $1/\tau$ values are plotted against $E^{\frac{1}{2}}$ values, initially. The points lie scattered in the field, as would be noted on a perusal. If, however, we plot log $1/\tau$ against $E_\alpha^{\frac{1}{2}}+0.018(90-Z)$, for these even-even nuclei, the points move over to a straight line, as has been indicated in the same figure. It is interesting to note that in the

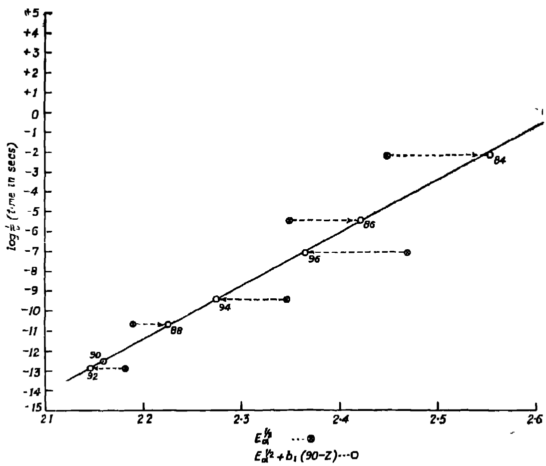

Fig. 6. $\mathrm{Log}_{10}\ 1/\tau$ Vs. $f(E_\alpha^{\frac{1}{2}})$ for $(N-Z)=50$.

case of all the even-even and odd-odd nuclei, excepting for the five low neutron content nuclei with $N \leqq 1.5Z$, an almost identical relation will bring the points corresponding to different $(N-Z)$ values, on different straight lines. Their inclinations are made to correspond, when the slopes are multiplied by the cubes of the respective $(N-Z)$ values. The displacement along the x-axis are measurable in terms of magnitudes which are nearly proportional to the $(N-Z)$ values, for all the different disintegration series. It puts all the available even-even and odd-odd nuclei excepting the low-neutron-content ones, into a simple linear relationship, to correlate the half lives with the α-energy and the neutron-proton content of the nucleus. In all, we have taken into consideration, now, thirty α-disintegrating nuclei, of which twenty-five are of the even-even type and five of the odd-odd type, whose data are complete. Their half lives are determined by the relation:

$$\log\frac{1}{\tau} = 2.117[(N-Z)^3\{E'_\alpha - b_i(90-Z)\}\times 10^{-4} + C_i(50-N-Z)] - 69.54 \quad \ldots \quad (7)$$

where $b_i = .018$ or $.009$ for even-even and odd-odd nuclei, $C_i = 1.750$, for $(N-Z)$ values from 54 to 48, which reduces to 1.725 and 1.625 for $(N-Z)$ as 46 and 44. τ is measured in seconds and E_α in Mev units. We have plotted in Fig. (7) the observed values of $\log 1/\tau$ against the function,

$$f(E_\alpha, N, Z) = \frac{(N-Z)^3}{50^3}[E'_\alpha - b_i(90-Z)] + C''_i(50-N-Z) \quad (7a)$$

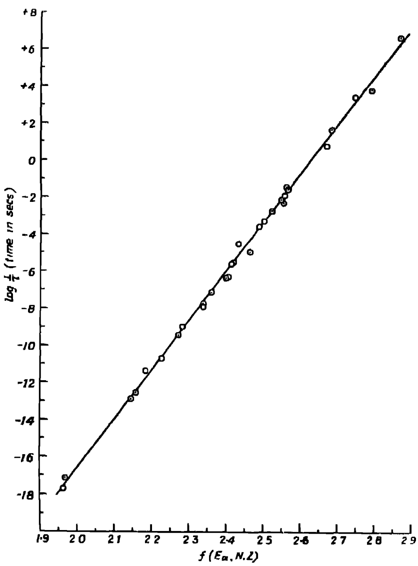

Fig. 7. $\text{Log}_{10}\ 1/\tau$ against calculated values of $f(E\alpha, N-Z)$ given by relation (7a).

where b_i's obtain the same values as before and C_i''s are 0.14, 0.138 and 0 130 in place of the above set of values for C_i. As a matter of fact, the indentity relation has been put on the basis of the graphical representation after the slope has been determined The calculated values of log $1/\tau$ for the 30 nuclei, have been tabulated in Table II, along with the observed log $1/\tau$ values and other relevant data.

The relations indicate clearly that, for a particular value of $(N-Z)$, the half lives of these nuclei decrease with increasing E_α values and with decreasing proton number For a particular E_α value, with the same proton number, however, the half life or log $1/\tau$ remains nearly balanced with increasing or decreasing excess neutron amount The additive term involving $(N-Z)$ almost balances the effect of the multiplying factor. Thus the more important factor in determining the half life is the α-energy amount and less predominantly, the proton number in the nucleus The increase in magnitude of the excess neutron would tend to decrease the half lives when the α-energy is high with a reverse effect when the α-energy is low. The half lives of these nuclei are dependent on the structural energy change 'S' only through the E_α-expression.

We hope to analyse the half-life data of the remaining even-odd and odd-even α-disintegrating nuclei as also of the low neutron content ones in a future work.

REFERENCES

Brussaard, P. J. and Thalhook, H A., 1958, *Physica*, **24**, 263.

Chang, W Y., 1946, *Phys. Rev*, **69**, 60 and 70, 632.

Dube, G. P and Singh, L. S, 1954, *Ind. J Phys*, **28**, 177.

Dutta, A, Pal, B., Dasgupta, A. and Chaudhury, N, 1962, *Ind. J. Phys.*, **36**, 497.

Evans, R D, 1955, The Atomic Nucleus, McGraw Hill Book Co. Inc. p. 529.

Everling, F, Konig, L. A., Mattauch, J. H. and Wapstra, A. H, 1960, *Nuclear Physics*, **18**, 519.

Fermi, E, 1951, *Nuclear Physics* (Chicago University Press), p 7.

Geiger, H and Nuttal, J. M, 1911, *Phil. Mag.*, **22**, 613.

Glueckauf, E, 1948, *Proc Phys. Soc* (*Lond*), **61**, 25.

Green, A. E S., 1954, *Phys. Rev.*, **95**, 1006.

Perlman, I., Ghiorso, A and Seaborg, G. T., 1950, *Phys. Rev.*, **77**, 26

Preston, M A, 1951, *Phys Rev*, **82**, 515.

Preston, M. A., 1962, Physics of the Nucleus (Addision Wesley Publishing Co.), p 349.

Ramaswamy, M K, 1956, *Nuovo Cimento*, **4**, 1570.

Strominger, D., Hollandar, J. N. and Seaborg, G. T., 1958, *Rev. mod. Phys.*, **30**, 585.

Suess, H. E and Jensen, J. H. D., 1952, *Arkiv F. Fysik*, **3**, 577.

Varshni, Y. P., 1956, *Nuovo Cimento*, **3**, 1148.

Way, K. and Wood, M, 1954, *Phys. Rev.*, **94**, 119.